Christ Church Grammar School

2019
TEST 2

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	3 minutes
Working time for this section:	30 minutes
Marks available:	26 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(5 marks)**

Find all the values, real and complex, of x for which $H(x) = 0$, if

$$H(x) = 10x^3 - 24x^2 + 10x - 4$$

$0 = 10x^3 - 24x^2 + 10x - 4 \quad \div 2$

$0 = 5x^3 - 12x^2 + 5x - 2$ ✓ simplifying expression

$0 = (x-2)(5x^2 - 2x + 1)$ ✓ Finding quadratic factor

$H(1) = 5 - 12 + 5 - 2$
$\neq 0$

$H(-1) = -5 - 12 - 5 - 2$
$\neq 0$

$H(2) = 40 - 48 + 10 - 2$
$= 0$

$\therefore (x-2)$ is a factor ✓ Finding real factor/ Solution

5	-12	5	-2	2
	10	-4	2	
5	-2	1	0	

$(x-2)(5x^2 - 2x + 1)$

Solve $5x^2 - 2x + 1 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 5 \times 1}}{10}$$

$$= \frac{2 \pm \sqrt{-16}}{10}$$

$$= \frac{2 \pm 4i}{10}$$

$$= \frac{1}{5} + \frac{2}{5}i$$

✓ using QF or completed square to solve quadratic $=0$

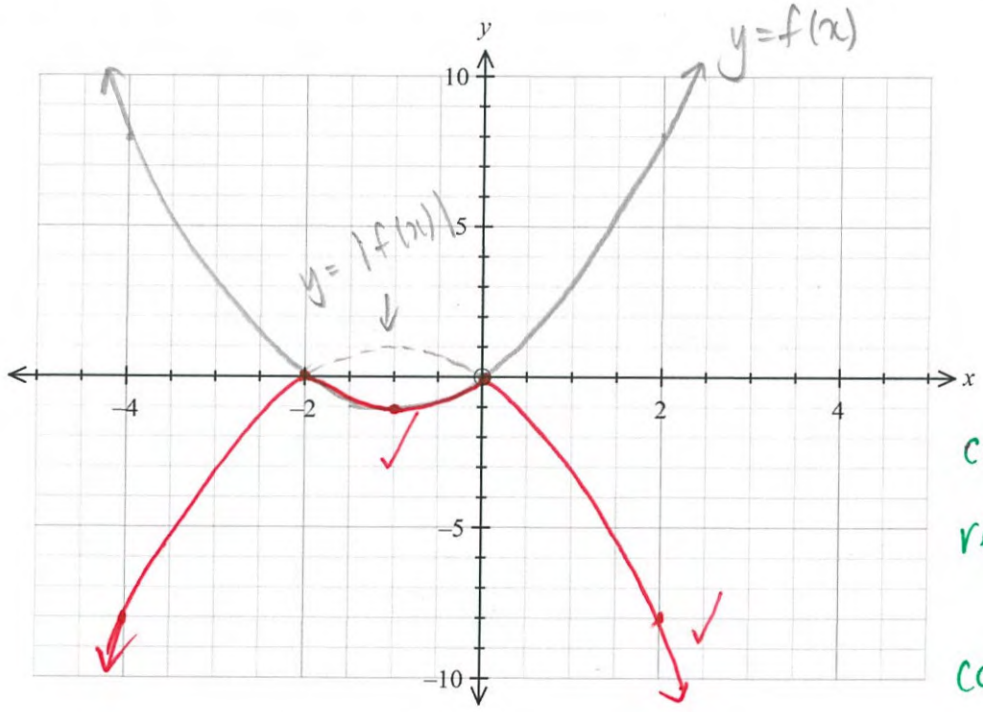
$\therefore$ Solutions are 2 and $\frac{1}{5} \pm \frac{2}{5}i$ ✓ stating solutions.

See next page

Question 2 **(8 marks)**

Consider the function $f(x) = x(x + 2)$.
Draw

(a) $y = -|f(x)|$. (3 marks)

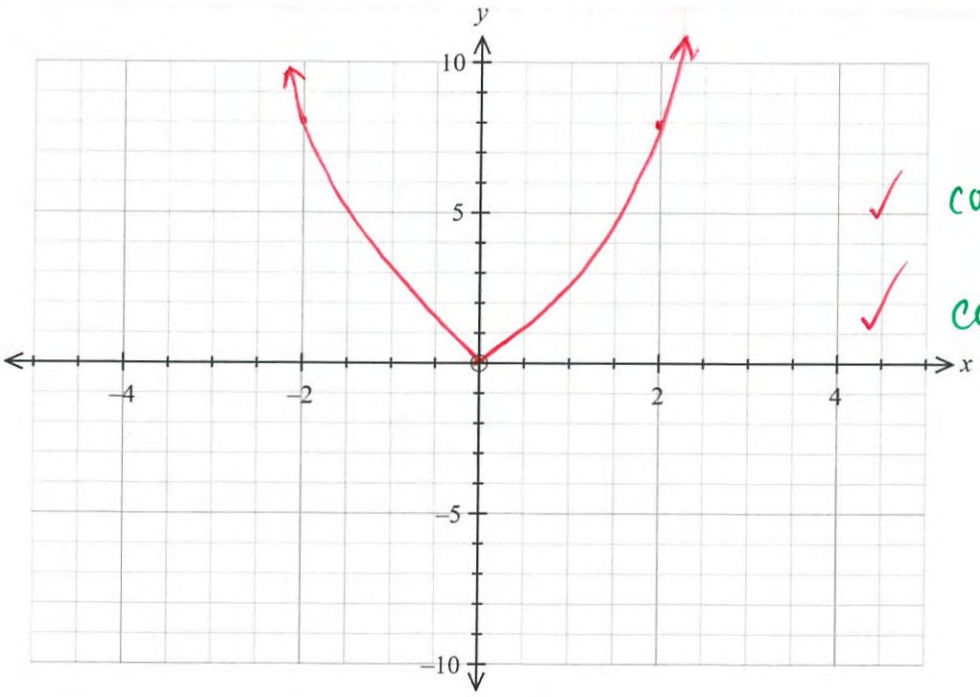


correct shape ✓

reflection over x-axis

correct absolute value ✓

(b) $y = f(|x|)$. (2 marks)



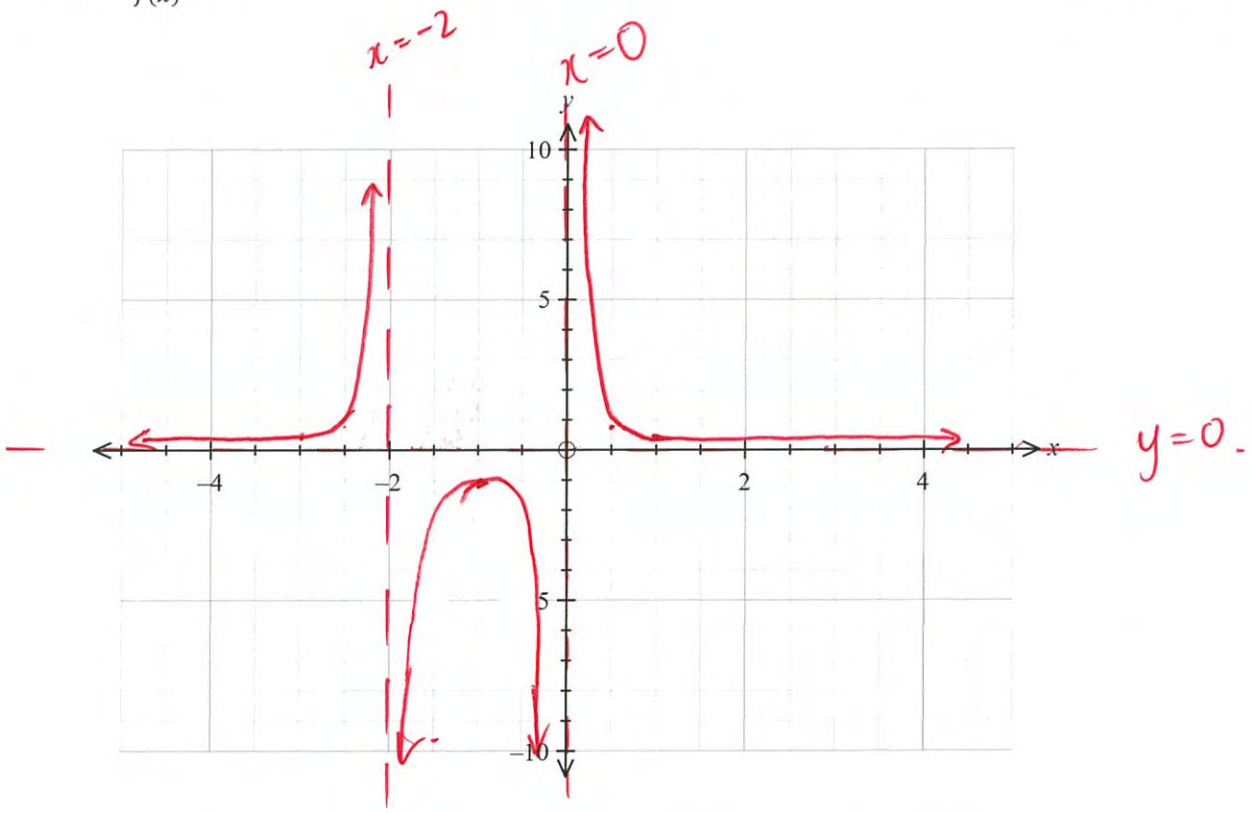
✓ correct shape

✓ correct $|x|$

See next page

Question 2 continued

(c) $y = \frac{1}{f(x)}$. (3 marks)



$y = \frac{1}{x(x+2)}$

$x = 1, y = \frac{1}{3}$

VA: $x = 0$, $x = -2$ ✓

HA: $y = 0$

asymptotes marked on graph.

TP $(-1, -1)$

$x \to 0^-, y \to -\infty$

$x \to -2^+ \; y \to -\infty$ ✓

correct on graph (shape of middle section + TP)

correct shape of graph (2 side parts)

$x \to 0^+, y \to \infty$

$x \to \infty, y \to 0^+$

$x \to -2^-, y \to \infty$

$x \to -\infty, y \to 0^+$

Question 3 (8 ~~9~~ marks)

If $f(x) = x^4 + 1$ and $g(x) = 5x - 4$, determine

(a) $g\,o\,f(x)$ (1 mark)

$= 5(x^4+1) - 4$ ✓ accept either solution.

$= 5x^4 + 1$

(b) the domain and range for $g\,o\,f(x)$. (2 marks)

$D_F \{x \in \mathbb{R}\}$ $R_F \{y \geq 1\}$

$D_G \{x \in \mathbb{R}\}$ $R_G \{y \in \mathbb{R}\}$

$\therefore$ Domain : $\{x \in \mathbb{R}:\}$ ✓ accept $\{y \geq 6\}$

Range : $\{y \in \mathbb{R} : y \geq 1\}$ ✓

(c) $f^{-1}(x)$, stating a suitable restriction to the domain of $f(x)$, so $f^{-1}(x)$ exists as a function. (3 marks)

$f^{-1}(x)$ will not exist unless $x \geq 0$ or $x > 0$ or $x \leq 0$ or $x < 0$ ✓ statement

$f(x) = x^4 + 1$

$y = x^4 + 1$

$\sqrt[4]{y-1} = x$ ✓ re-arrangement

Note if $x \leq 0$ or $x < 0$ then $x = -\sqrt[4]{y-1}$

$\therefore f^{-1}(x) = \sqrt[4]{x-1}$ ✓, when $x \geq 1$ correct $f^{-1}(x)$

or $f^{-1}(x) = -\sqrt[4]{x-1}$, $x \leq 1$

(d) the domain and range for $f^{-1}(x)$, such that $f^{-1}(x)$ is a function (2 marks)

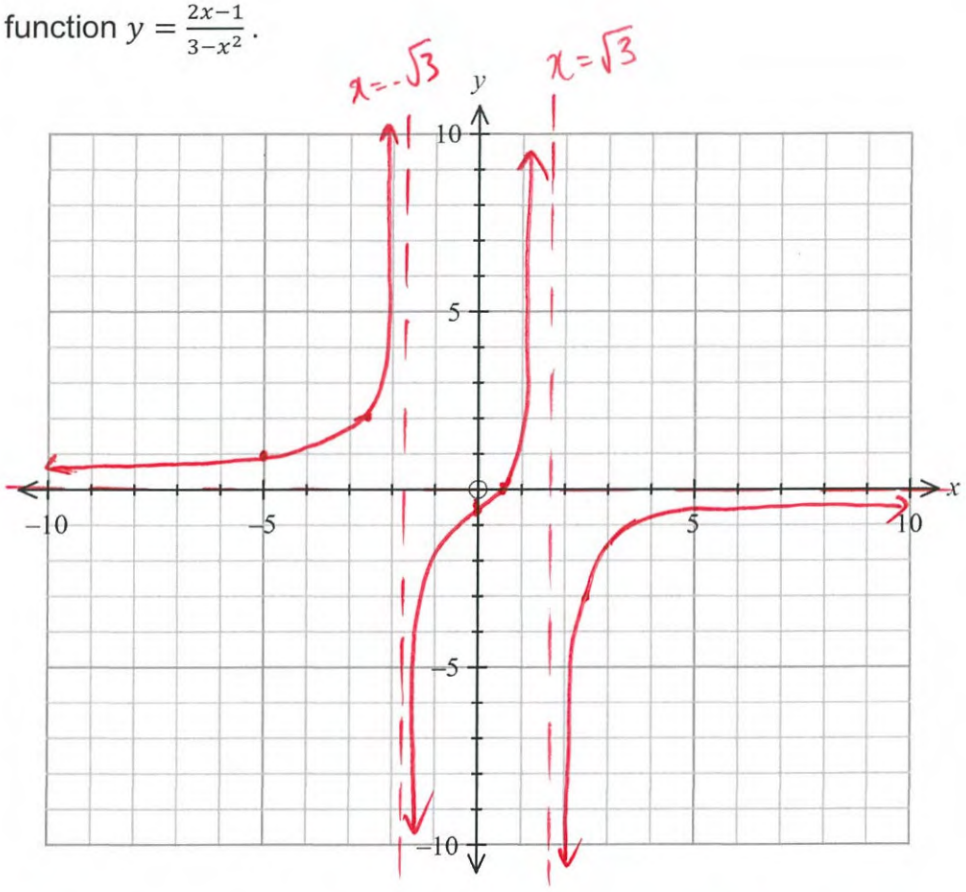
$D = \{x \in \mathbb{R} : x \geq 1\}$ ✓

$R : \{y \in \mathbb{R} : y \geq 0\}$ ✓

↑ this may change depending on restriction from part (c).

Question 4 **(5 marks)**

Draw the function $y = \frac{2x-1}{3-x^2}$.



VA: $3-x^2=0$

$x = \pm\sqrt{3}$. ✓ correct asymptotes (labelled on graph)

HA: $y=0$

$x=0, y=-\frac{1}{3}$

$y=0, x=\frac{1}{2}$ ✓ x-intercept + y-intercept correct

$x \to \sqrt{3}^- \quad y \to \infty$

$x \to -\sqrt{3}^+ \quad y \to -\infty$ ✓

$x \to \sqrt{3}^+ \quad y \to -\infty$

$x \to \infty \quad y \to 0^-$ ✓

$x \to -\sqrt{3}^- \quad y \to \infty$

$x \to -\infty \quad y \to 0^+$ ✓

correct shape for each section.

no t.ps.

evident on graph working not required as such.

End of questions

Christ Church Grammar School

2019
TEST 2

MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

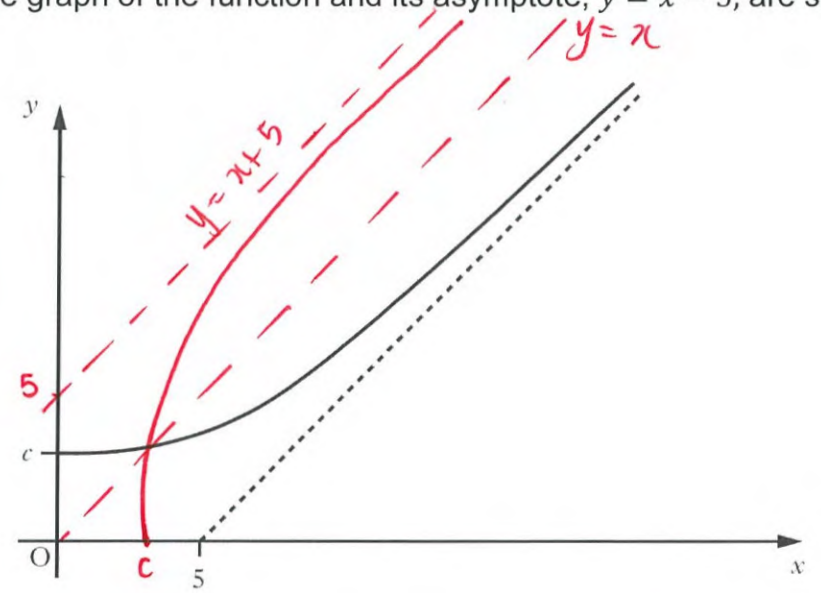
6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 5 **(6 marks)**

The function $f(x)$ is defined for all $x \geq 0$.
The graph of $y = f(x)$ intersects the $y - axis$ at $(0, c)$, where $0 < c < 5$.
The graph of the function and its asymptote, $y = x - 5$, are shown below.



(a) On the diagram above, draw $f^{-1}(x)$. Clearly label any points of intersection and any asymptotes. (4 marks)

✓ shape and behaviour of graph as it approaches asymptote
✓ asymptote $(y = x + 5)$, parallel $y = x - 5$
✓ 5 marked on y-axis at asymptote and 'c' on x-axis.
✓ Shape symmetrical

(b) What is the equation of the asymptote of the graph of $y = f(x + 2)$? (1 mark)

$y = x - 3$ ✓ $\quad (y = (x+2) - 5 = x - 3)$

(c) Why does your diagram show that the equation of $x = f^{-1}(f(x))$ has at least one solution? (1 mark)

$y = f(x)$ intersects at $y = x$ so $x = f^{-1}(f(x))$.

or $y = f^{-1}(x)$ intersects at $y = x$ so $x = f^{-1}(f(x))$. ✓ valid reason from observation of graph or algebraic reason.

or $f^{-1}(x) = f(x)$ so $x = f^{-1}(f(x))$.

See next page

Question 6 **(5 marks)**

Let the function $f(x)$ be given by

$$f(x) = \frac{2x^3 - 7x^2 + 4x + 5}{(x-2)^2}, x \neq 2.$$

(a) The graph of $y = f(x)$ crosses the $y - axis$ at $(0, a)$. State the value of a. (1 mark)

$x = 0 \quad f(0) = \frac{5}{(-2)^2} = \frac{5}{4}$ $\therefore a = \frac{5}{4}$ ✓ explicitly states $a =$

For the graph of $y = f(x)$,

(b) (i) write down the value of the vertical asymptote. (1 mark)

$x = 2$ ✓

(ii) show algebraically that there is a non-vertical asymptote and states its equation. (3 marks)

NVA : $(x-2)^2 = x^2 - 4x + 4$ ✓ expands brackets

$$\begin{array}{r} 2x+1 \\ x^2-4x+4 \;\overline{)\; 2x^3-7x^2+4x+5} \\ -(2x^3-8x^2+8x) \\ \hline x^2-4x+5 \\ -(x^2-4x+4) \\ \hline 1 \end{array}$$

✓ performs division

$f(x) = 2x + 1 + \frac{1}{x^2-4x+4}$

$\therefore$ NVA is $y = 2x + 1$. ✓ states NVA.

See next page

Question 7 **(4 marks)**

If $-3 + i$ is a solution of $P(x) = ax^3 + 9x^2 + ax - 30$, where a is real, determine the value of a and hence find all zeros of the cubic.

If $-3+i$ is a solution then $-3-i$ ✓ is a solution

identifies conjugate as another solution

$(-3-i)(-3+i)$

$= 9+1$

$= 10$

$\therefore x = 10$

$+ = -6.$

$(x+3\mp i)(x+3+i)(ax+b) = ax^3+9x^2+ax-30$

$\Rightarrow (x^2+6x+10)(ax+b) = ax^3+9x^2+ax-30$ ✓

correctly finds quadratic bracket $x^2+6x+10$

$\therefore b = -3$

$(x^2+6x+10)(ax-3) = ax^2+9x^2+ax-30$

x^2: $9 = 6a-3$

$a = 2$

or x: $a = 10a-18$

$a = 2$ ✓ Solves for $a=$

$(x^2+6x+10)(2x-3) = 0$

So solutions

$x = -3 \pm i, \frac{3}{2}$ ✓ states solutions.